

Factor supply elasticity of factor price given a Cobb–Douglas production function

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October 2026

An example of the practical relevance of the present exercise is the assessment of the macroeconomic gains from immigration accruing to the resident population, the so-called immigration surplus, and the associated redistributive effects.¹

The Cobb–Douglas production function

We assume the following textbook Cobb–Douglas production function:

$$y = kx_1^\alpha x_2^{1-\alpha} \quad (k > 0, 0 < \alpha < 1) \quad (1)$$

where y is total output, k is a constant, and x_1, x_2 are the inputs of factors of production 1 and 2 (e.g. labour and capital). The coefficient α is the output elasticity with respect to factor 1, and likewise $(1-\alpha)$ is the output elasticity with respect to factor 2. Since $\alpha+(1-\alpha)=1$, the production function in equation (1) implies constant returns to scale.

We further assume that the supplies of factors of production 1 and 2 are inelastic with respect to price. A clearing market for factor 1 implies equality between its price and its marginal productivity:

$$p_1 = \frac{\partial y}{\partial x_1} = \alpha k x_1^{\alpha-1} x_2^{1-\alpha} \quad (2)$$

From equations (1) and (2) it follows that:

$$p_1 = \frac{\alpha y}{x_1} \quad (2a)$$

Note that the income accruing to factor of production 1 equals a fraction α of output y :

$$x_1 p_1 = \alpha y \quad (2b)$$

Analogously, for factor of production 2 we obtain:

$$x_2 p_2 = (1 - \alpha)y \quad (2c)$$

Thus, the factor payments correspond to their respective shares in output, implying that total factor payments exhaust total revenue.

¹ Borjas, G. J. (1999), *The economic analysis of immigration*, in: Ashenfelter, O., Card, D. (Eds.), *Handbook of Labor Economics*, Vol. 3, North-Holland, Amsterdam, pp. 1697–1760; Euwals, R., Roodenburg, H., *A Note on the Redistributive Effect of Immigration*, *Economics Letters*, Vol. 85, issue 2, pp. 241–246, 2004

Derivation of the elasticities

The factor supply elasticity of factor price, ε_{ij} , is defined as:

$$\varepsilon_{ij} \equiv \frac{\partial(\log p_i)}{\partial(\log x_j)} \quad (3)$$

Here, p_i denotes the price of factor i , and x_j denotes the input of factor j .

For small changes, the numerator can be approximated as:

$$\partial(\log p_i) \approx \frac{1}{p_i} \cdot \partial p_i \quad (4)$$

Similarly, the denominator can be approximated as:

$$\partial(\log x_j) \approx \frac{1}{x_j} \cdot \partial x_j \quad (5)$$

Substituting these expressions for factor of production 1 yields:

$$\varepsilon_{11} \approx \frac{\partial p_1}{\partial x_1} \cdot \frac{x_1}{p_1} \quad (6)$$

Differentiating equation (2) with respect to x_1 yields:

$$\frac{\partial p_1}{\partial x_1} = (\alpha - 1)\alpha y x_1^{-2} \quad (7)$$

Substitution of equations (7) and (2a) reduces equation (6) to:

$$\varepsilon_{11} \approx \alpha - 1 \quad (8)$$

Analogously, we obtain for factor of production 2:

$$\varepsilon_{22} \approx -\alpha \quad (9)$$

Thus, given a Cobb–Douglas production function, the own-factor supply elasticity of factor price is constant and equals the corresponding output elasticity minus one. If factor of production 1 is assumed to be labour, the elasticity ε_{11} is required in estimating the immigration surplus.

Applying a similar procedure, the cross-elasticities are:

$$\varepsilon_{12} \approx 1 - \alpha \quad (10)$$

$$\varepsilon_{21} \approx \alpha \quad (11)$$

Elasticity ε_{21} is required for estimating the redistributive effects associated with the immigration surplus.